2.1 The Pythagorean Theorem

Pythagoras (c. 530 B.C.E.) was a mathematician, philosopher, and astronomer. He believed that everything could be predicted and measured in patterns or cycles. Pythagoras is credited with one of the most useful relationships in mathematics, the **Pythagorean theorem**. The Pythagorean theorem is useful when building structures, such as the International Space Station.

Do you think Pythagoras ever imagined we would still be using the Pythagorean theorem more than 2500 years later ... and in space?

Investigate

The Pythagorean Relationship

Tools

- 1-cm grid paper
- coloured paper
- protractor
- ruler
- scissors
- tape

1. On 1-cm grid paper, draw the three squares in each set. Label the squares with the set number, then cut the squares out. Find the area of each square in square centimetres, and record the area on the square.
   - **Set 1** squares with each side length: 3 cm, 4 cm, and 5 cm
   - **Set 2** squares with each side length: 5 cm, 12 cm, and 13 cm
   - **Set 3** squares with each side length: 6 cm, 8 cm, and 10 cm
   - **Set 4** squares with each side length: 4 cm, 5 cm, and 6 cm
   - **Set 5** squares with each side length: 4 cm, 12 cm, and 14 cm
2. Arrange the squares in each set to form a triangle. One side of each square forms one side of the triangle. Tape each arrangement onto a piece of coloured paper.

Math Connect

A theorem is a mathematical statement that has been proven.

3. Use a protractor to determine whether any of the triangles contain a right angle.

4. Copy and complete the table based on your observations of the five triangles.

Set	Side Lengths (cm)			Areas of Squares (cm^2)			Type of Triangle (right, acute, or obtuse)
1	3	4	5				
2	5	12	13				
3	6	8	10				
4	4	5	6				
5	4	12	14				

5. Look for a pattern in your results. Compare the areas of the squares on the sides of each triangle. Write a sentence to describe the relationship.

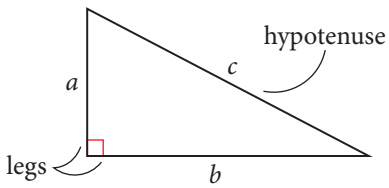
6. Look at the last two triangles. Does this relationship hold true for these triangles? How are these two triangles different from the first three?

legs
- the two shorter sides of a right triangle
- the sides adjacent to the right angle

hypotenuse
- the longest side of a right triangle
- the side opposite the right angle

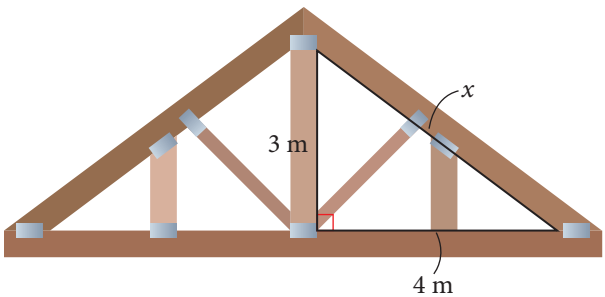
Pythagorean theorem
- the square of the hypotenuse is equal to the sum of the squares of the legs
- for a right triangle with legs a and b and hypotenuse c, $c^2 = a^2 + b^2$

In a right triangle, the two shorter sides are called the **legs**. The **hypotenuse** is the side opposite the right angle; it is the longest side. The **Pythagorean theorem** states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.



Example 1 Find the Length of a Roof Truss

Find the length of a slanting side of this roof truss.

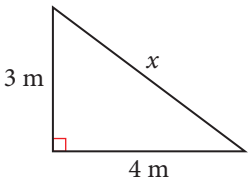


Math Connect

A roof truss uses a series of triangles secured together to support the roof of a house, or other building. Trusses are usually premade of lumber fastened with metal plates.

Solution

Part of the roof truss is in the shape of a right triangle. The lengths of the legs are 3 m and 4 m. Use the Pythagorean theorem to find the length of the hypotenuse.



$x^2 = 3^2 + 4^2$
$x^2 = 9 + 16$
$x^2 = 25$
$x = 5$

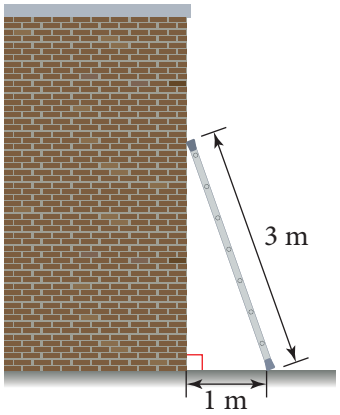
The slanting side of the roof truss has a length of 5 m.

Example 2 Find How High a Ladder Will Reach

Nirmala is planning to wash the windows on her house. She rests a 3-m-long ladder against the side of the house. The foot of the ladder is 1 m from the house. How high up the house will the ladder reach? Round your answer to the nearest hundredth of a metre.

Solution

Draw a diagram to represent the situation.

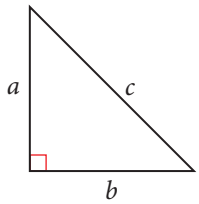


$3^2 = 1^2 + h^2$
$1 + h^2 = 9$
$h^2 = 9 - 1$
$h^2 = 8$
$h = \sqrt{8}$
$h \doteq 2.828$

The ladder will reach approximately 2.83 m up the side of the house.

Key Concepts

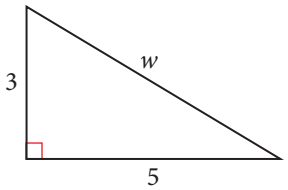
- In a right triangle, the hypotenuse is the longest side.
- The square of the hypotenuse is equal to the sum of the squares of the legs.
- You can use the Pythagorean theorem to find the length of one side of a right triangle given the lengths of the other two sides.



$c^2 = a^2 + b^2$

Discuss the Concepts

D1. Explain how you would find the length of w. Compare your answer with that of a classmate.

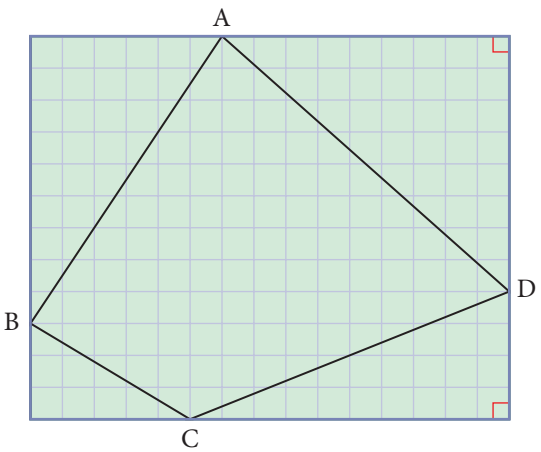


D2. When you look at a right triangle, how can you tell which two sides are the legs? How can you tell which side is the hypotenuse?

Practise the Concepts A

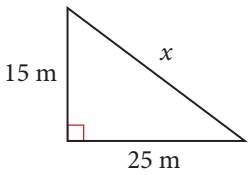
For help with questions 1 and 2, refer to Example 1.

1. Find the length of each hypotenuse. Round your answers to one decimal place.



2. Find the length of the hypotenuse to the nearest tenth of a unit.

a)



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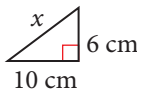

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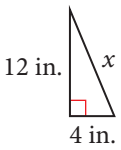

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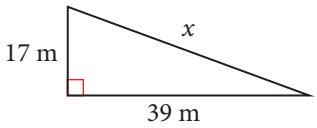


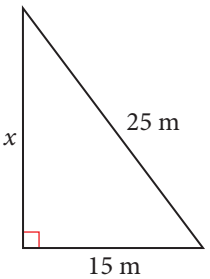
Math Connect

Watch an animation proving the Pythagorean Theorem. Then, demonstrate the Pythagorean Theorem yourself. Go to www.mcgrawhill.ca/links/foundations10 and follow the links.

For help with questions 3 and 4, refer to Example 2.

3. Find the length of the indicated side to the nearest tenth of a metre.

a)



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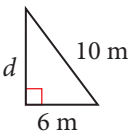

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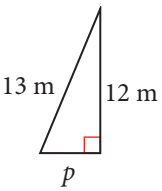

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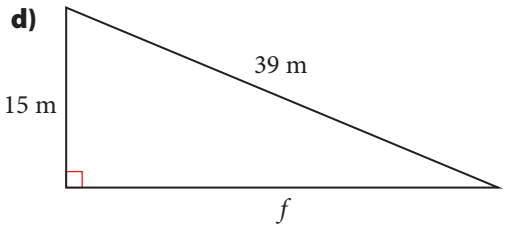
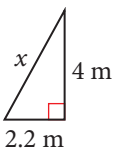


4. Find the length of x to the nearest tenth of a metre. Compare your answers with those of a classmate. Did you both get the same answers? Is each answer reasonable?

a)



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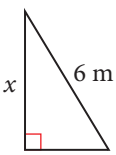

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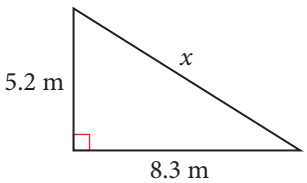


5. Jie-ling walks home from school by walking around two sides of a rectangular park. The length of the park is 125 m and the width is 121 m. If Jie-ling were to walk diagonally across the park, how far would she walk?

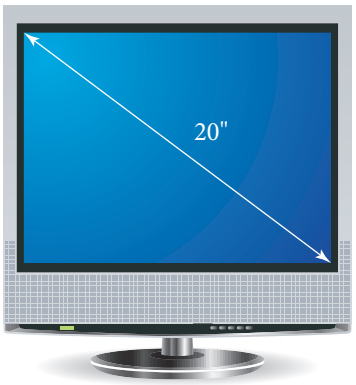
Apply the Concepts B

6. a) The school has to build a wheelchair ramp outside the front doors. The current stairs go through a vertical rise of 1 m. If the ramp is to be 14.6 m long, how far from the school will the ramp start? Round your answer to the nearest tenth of a metre.

b) If there is a wheelchair ramp at your school, with a partner, measure the length of the ramp and its vertical height. Use the Pythagorean theorem to calculate the horizontal distance from the school to the end of the ramp. Then, measure the horizontal distance and compare it to your calculated result.

Literacy Connect

7. A television is described as a 20" television if the screen has a diagonal length of 20".



a) If the screen of a 20" flat-screen television has a height of 12", what is the width?

b) If a new 55" plasma television screen has a height of 35", what is the width of the screen?

c) Do you feel the practice of sizing a television by referring to the diagonal width of its screen is a fair one? Explain.

d) What misconceptions might a customer have when going to buy a television?

e) Explain why you think this practice was started. Do you think it benefits the manufacturer, the retail storeowner, or the customer? Explain your reasoning.

f) In small groups, discuss your answers.

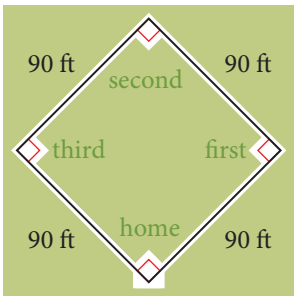
8. Samir is finishing his basement. He needs sheets of drywall that are 4' by 8'. His basement doorway is 85" tall and 36" wide. There is a landing at the top of the basement stairs, so Samir must carry the drywall with the longer side up. Will the drywall fit through the basement door? How do you know?

9. Sam is quilting a table runner of length 38 cm and width 22 cm. Sam will stitch along the diagonal, and then stitch lines parallel to the diagonal.

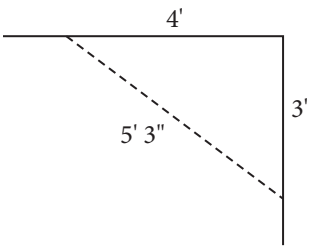
a) To the nearest centimetre, how long is the diagonal?

b) Build a model of Sam's table runner. Measure the diagonal on your model. Compare the value with the one you calculated in part a) above.

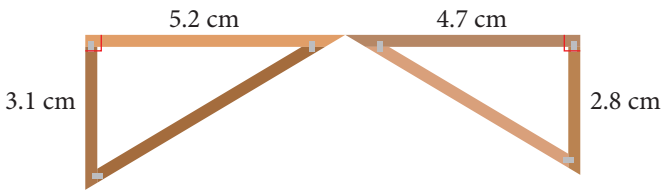
10. Natalya is playing baseball. She catches a ground ball at third base. The player on the opposing team is running toward first base. How far does Natalya have to throw the ball to throw the runner out?



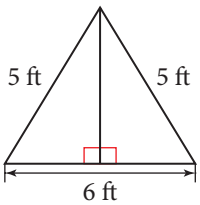
11. Sue and Greg are laying new laminate flooring in their living room. To check that the walls are square, Sue makes marks 3' from the corner along one wall and 4' from the same corner along the other wall. She measures the distance between the marks to be 5' 3". Do the walls in this corner meet at right angles? How do you know?



12. Daniel is building a wooden bridge for his daughter's model railroad. He sketches a plan for the bridge. What length of wood does Daniel need to build the bridge?

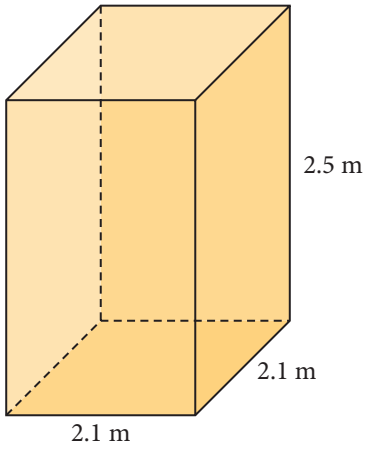


13. Darlene goes camping with her children. As they set up the tent, they discover that the vertical support poles are missing. What length of pole does Darlene need to buy?



Chapter Problem

14. Jeff wants to make the cage portion of the elevator (the part in which passengers will ride). He knows he will have to brace each side of the cage.



a) If each side of the cage is 2.1 m wide by 2.5 m high, how long will the brace piece be if Jeff wants the brace to go the full diagonal of the side?

b) The base of the cage will be 2.1 m square. What length will the diagonal brace piece for the base of the cage be?

Extend the Concepts C

15. A loading ramp is 2.8 m long. One end rests on a loading dock 0.7 m above the ground, and the other end leads into the back of a tractor trailer 1.2 m above the ground. Find the horizontal distance between the back of the truck and the loading dock, to the nearest tenth of a metre.